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Photon interferometry of quark-gluon dynamics revisited

A. Timmermann^{*}, M. Plümer[†], L. Razumov[‡] and R.M. Weiner[§]

Physics Department, Univ. of Marburg, Marburg, FRG

Abstract

The Bose-Einstein correlations of photons emitted from a longitudinally expanding system of excited matter produced in ultrarelativistic heavy ion collision are studied. Two effects found in recent calculations – that the correlation function in longitudinal direction exhibits oscillations, and that it takes values below unity – are demonstrated to be numerical artefacts and/or the results of inappropriate approximations. Thus, the general quantum statistical bounds for the two particle correlation function of a chaotic source with Gaussian fluctuations are confirmed. Two different expressions for the two-photon inclusive distribution are considered. Depending on which of the two expressions is used in the calculation, the width of the correlation function may vary by as much as 30%.

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^{*}E. Mail: TIMMERMA@CONVEX.HRZ.UNI-MARBURG.DE

[†]E. Mail: PLUEMER_M@VAX.HRZ.UNI-MARBURG.DE

[‡]E. Mail: RAZUMOV@CONVEX.HRZ.UNI-MARBURG.DE

[§]E. Mail: WEINER@VAX.HRZ.UNI-MARBURG.DE

The study of high energy photons can yield information about the early stages of the evolution of hot and dense matter created in ultrarelativistic nuclear collisions. This is of special interest in the experimental search for a quark-gluon-plasma (QGP), a new phase of matter expected to exist for a brief period of a few fm/c before the majority of the final state hadrons are emitted. In particular, information on the space-time extension of the excited matter which can be extracted from the two-photon correlation function may be used to determine the energy densities reached in the collision.

Under the title “Photon interferometry of quark-gluon dynamics”, a calculation of the Bose-Einstein correlation (BEC) function of photons emitted from a system of longitudinally expanding hot and dense matter at RHIC and LHC energies was presented in Ref. [1]. The results were later extended to include the effects of transverse flow, i.e., to the case of a full three-dimensional hydrodynamic expansion [2, 3]. It was argued[3] that the BEC’s of photons of high transverse momenta (on the order of a few GeV/fm) can be sensitive to the presence of a mixed phase and may thus provide a signature for the QGP.

It is the purpose of this note to point out that two rather surprising features of the two-photon correlation function presented in Ref. [1] do not reflect genuine properties of the source. Rather, they are artefacts of inappropriate approximations used to evaluate space-time integrals in [1]. To be specific, in Ref. [1] it was found that the BEC function in longitudinal direction (a) exhibits oscillations and (b) takes values below unity. As was discussed in Ref. [4], property (b) is inconsistent with the quantum statistical bounds for a purely chaotic source. Below, it will be demonstrated that both properties (a) and (b) disappear if the integrals are calculated more exactly.

The BEC function of two identical particles can be written as

$$C_2(\vec{k}_1, \vec{k}_2) = \frac{P_2(\vec{k}_1, \vec{k}_2)}{P_1(\vec{k}_1)P_1(\vec{k}_2)}, \quad (1)$$

where \vec{k}_i ($i = 1, 2$) are the three-momenta of the particles, and $P_2(\vec{k}_1, \vec{k}_2)$ and $P_1(\vec{k})$ are the one- and two-particle inclusive spectra. For the general case of a Gaussian density matrix and a purely chaotic source, the current formalism allows to relate all m -particle inclusive distributions to a source function $w(x, k)$ which describes the mean number of particles of four-momentum k emitted from a source element centered at the space-time point x (cf. [4] and refs. therein). For the one- and two-particle spectra, one has

$$P_1(\vec{k}) = \int d^4x w(x, k), \quad (2)$$

and

$$P_2(\vec{k}_1, \vec{k}_2) = P_1(\vec{k}_1)P_1(\vec{k}_2) + \int d^4x_1 d^4x_2 w\left(x_1, \frac{k_1 + k_2}{2}\right) w\left(x_2, \frac{k_1 + k_2}{2}\right) \cos(\Delta x^\mu \Delta k_\mu). \quad (3)$$

The expressions (2) and (3), which are usually applied in calculations of the BEC function, have also been derived in a Wigner function approach in [5]. In Ref. [1], however, a different form was used for the two-particle inclusive distribution, namely,

$$P_2(\vec{k}_1, \vec{k}_2) = P_1(\vec{k}_1)P_2(\vec{k}_2) + \int d^4x_1 d^4x_2 w(x_1, k_1) w(x_2, k_2) \cos(\Delta x^\mu \Delta k_\mu). \quad (4)$$

The form (4) was also derived in Refs. [6] as an approximation to the complete, non-analytic result. As was shown in [4], the expression (4) has the disadvantage that in certain cases it can lead to results for BEC function which violate the quantum statistical bounds for a purely chaotic source,

$$1 \leq C_2(\vec{k}_1, \vec{k}_2) \leq 2, \quad (5)$$

whereas (3) automatically satisfies Eq. (5).

We have evaluated the above expressions (3) and (4) for the case of thermal photons emitted from a longitudinally expanding system of excited matter created in $Pb + Pb$ collisions at $\sqrt{s} = 200$ AGeV. We use the source function [7] that was adopted in [1],

$$w(x, k) = \text{const. } T(x)^2 \ln \left(\frac{2.9 k_\mu u^\mu(x)}{g^2 T(x)} + 1 \right) \exp \left(-\frac{k_\mu u^\mu(x)}{T(x)} \right) \quad (6)$$

where g is the QCD coupling constant, $T(x)$ is the local temperature and $u^\mu(x)$ the local flow velocity. To allow for comparison of our results with those of Ref. [1], we used the same model for the space-time evolution of the dense matter as in [1]. That is to say, we applied Bjorken hydrodynamics [8] and adopted a massless pion gas equation of state and a bag model equation of state for the hadronic phase and the QGP phase, respectively. The results presented below correspond to an initial proper time $\tau_i = 0.124$ fm/c, an initial temperature $T_i = 532$ MeV and a freeze-out temperature $T_f = 140$ MeV.

Fig. 1 shows the BEC function¹ in longitudinal direction, i.e., for a configuration of equal transverse momenta, $\vec{k}_{1\perp} = \vec{k}_{2\perp}$. The correlation functions are plotted against the rapidity difference $\Delta y = y_1 - y_2$. The results presented in Fig. 1 refer to photons emitted from the QGP phase only, at a transverse momentum $k_\perp = 5$ GeV/c.

The curves labeled 1 and 2 were obtained by performing all space-time integrations numerically. Curve 1 corresponds to the expression (3) and curve 2 to the expression (4) for $P_2(\vec{k}_1, \vec{k}_2)$. Neither of the two results shows oscillatory behaviour, and they both respect the quantum statistical bounds (5). It is, however, interesting to note that the width of the correlation function increases by about 30% if (4) rather than (3) is used

¹Here and in the following, we consider pairs of photons of equal polarization. Averaging over polarizations leads to modifications which manifest themselves, among other things, in a decrease of the intercept of the correlation function.

in the calculation. Let us now consider the approximation adopted in Ref. [1]. The space-time integrals in (2) – (4) involve integrations over the space-time rapidities η_1 and η_2 . In [1], the latter were performed analytically by using a Gaussian approximation of the integrand as a function of η_1 and η_2 in Eq. (4). This procedure leads to the results displayed as curve 3 in Fig. 1. The corresponding correlation function oscillates as a function of Δy and even takes values below one, i.e., it violates the quantum statistical bounds (5). A comparison with the “exact” results (curve 2) shows that the Gaussian approximation is inappropriate in this case. For completeness, we have also included the cosine-approximation proposed in [1] for small Δy (curve 4 in Fig. 1),

$$C_2(\vec{k}_1, \vec{k}_2) \approx 1 + \cos[4k_\perp \tau_i \sinh^2(\Delta y/2)]. \quad (7)$$

In Fig. 2 the BEC in longitudinal direction is shown for the sum of all three contributions (hadron gas, mixed phase and QGP). As was already observed for the QGP component alone, at $k_\perp = 5 \text{ GeV}$ the curves obtained by using the expressions (3) and (4) differ in width by a factor of ~ 1.3 . On the other hand, the corresponding two curves for $k_\perp = 2 \text{ GeV}$ are almost indistinguishable. It is noteworthy that while the 5 GeV curves are of approximately Gaussian shape, the 2 GeV curves are distorted and suggest a two (or more) component structure. Fig. 3 compares the plasma contributions to the sum of the contributions from all phases. For 5 GeV photons the plasma contribution dominates whereas for transverse momenta of 2 GeV the shapes of C_2 for the plasma component and the sum of all components differ strongly. Indeed, as can be seen in Fig. 4 the transition from a two-component shape to a Gaussian shape occurs in a narrow range of transverse momenta, $2 \text{ GeV} \leq k_\perp \leq 3 \text{ GeV}$. This behaviour can be understood in terms of the relative importance of the contributions from the QGP, mixed and hadronic phase. Fig. 5 shows the production rate of photons at $y = 0$ as a function of longitudinal proper time τ for five different values of k_\perp . The two points in proper time where the slope changes discontinuously correspond to the transition from the QGP to the mixed phase and from the mixed phase to the hadron gas, respectively. For high transverse momentum photons (e.g., $k_\perp = 4 \text{ GeV}$), the QGP contribution dominates, and consequently, there appears a single Gaussian in the correlation function (cf. Fig. 3). On the other hand, for smaller transverse momenta ($k_\perp \sim 1 \text{ GeV}$) the contribution from the mixed phase becomes comparable to or even exceeds the plasma contribution, resulting in a two-component structure of C_2 . Thus, if such a change of shape as a function of k_\perp will be observed it may be considered as a piece of evidence for a first order phase transition.²

²Of course, other effects can also give rise to a two-component structure. One important example is quantum statistical partial coherence [9].

In [3], it was argued that a two-component form of the two-photon correlation function in *transverse* direction may signify the presence of a mixed phase and hence, of a first order phase transition. In Fig. 6 the BEC function is plotted against the component q_{out} of the transverse momentum difference (i.e., the component parallel to the transverse momentum of the pair), for various values of $k_{1\perp}$. The results displayed in the figure which were obtained by using the expression (3) roughly agree with those obtained in Ref. [1] by using (4). As in the case of the correlation function in longitudinal direction, one observes the deviation from the Gaussian shape as k_{\perp} decreases, which signifies the increasing importance of the contribution from the mixed phase. Note that in contrast to the BEC function in Δy , the correlation function in q_{out} does show oscillations. However, it does not take values below unity, i.e., it does not violate the quantum statistical bounds (5).

To summarize, we have demonstrated that two rather surprising properties of the two photon correlation function presented in a recent publication are artefacts of inappropriate approximations in the evaluation of space-time integrals. In [1], it was found that the BEC function in longitudinal direction (a) oscillates and (b) takes values below unity. As property (b) is inconsistent with general statistical bounds, it is important to clarify the origin of this discrepancy. We have shown that both properties (a) and (b) disappear if the space-time integrations are performed numerically (i.e., without adopting any analytic approximation). On the other hand, we have confirmed that the correlation function in transverse direction does exhibit oscillatory behaviour in the *out*-component of the momentum difference, as was observed in Ref. [1]. We have considered two different forms for the two-particle inclusive distribution and found that the widths of the resultant correlation functions may differ by up to 30% depending on which one of the two expressions is used in the calculation. The change of the BEC function in Δy from a Gaussian to a two-component shape with decreasing transverse momentum of the pair may serve as evidence for the presence of a mixed phase, and hence, of a first order phase transition.

Note that the results discussed above refer to a purely longitudinal expansion. While there are indications[10] that at high collision energies the expansion indeed becomes effectively one-dimensional, it will be interesting to extend the present considerations to the case of a fully three-dimensional hydrodynamic flow. However, this is a task which exceeds the scope of this brief report.

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Figure Captions

Fig. 1 Two-photon Bose-Einstein correlation C_2 as a function of the rapidity difference Δy , for photons of transverse momentum $k_\perp = 5 \text{ GeV}$ emitted in the QGP. The curves correspond to different expressions and approximations for the two-photon inclusive distribution.

Fig. 2 Comparison of $C_2(\Delta y)$ calculated from the two expressions (3) and (4) (see text). The results include contributions from all three components (QGP, mixed phase and hadron gas).

Fig. 3 Comparison of the complete correlation function $C_2(\Delta y)$ and the separate contribution from the QGP phase.

Fig. 4 Dependence of the BEC function $C_2(\Delta y)$ on the transverse momentum k_\perp of the pair.

Fig. 5 Dependence of the production rate of thermal photons on the longitudinal proper time, for different values of the transverse momentum.

Fig. 6 Two-photon BEC function in transverse direction, as a function of the momentum difference in *out*-direction, for different values of the transverse momentum $k_{1\perp}$ of one of the two photons.

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Fig. 1

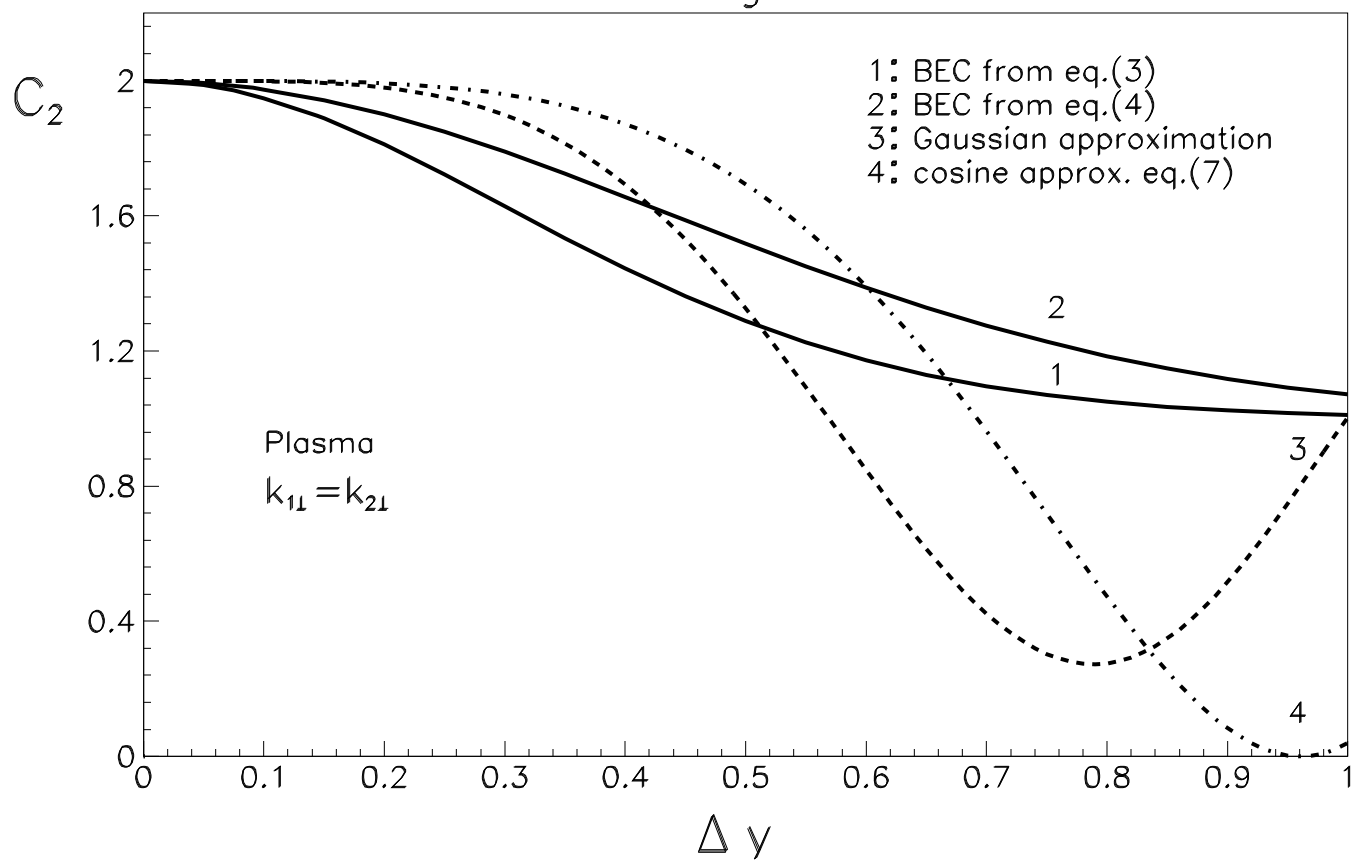


Fig. 2

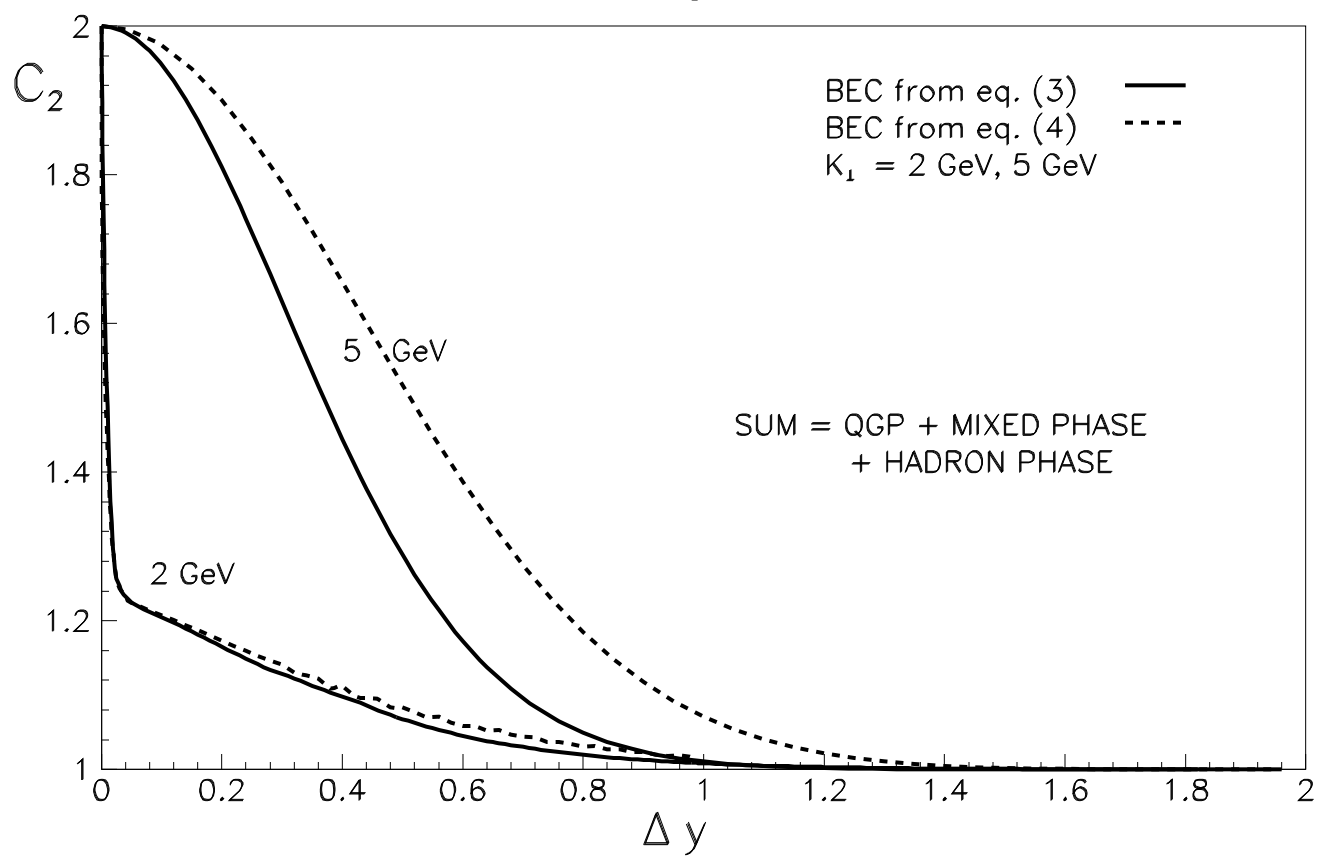


Fig. 3

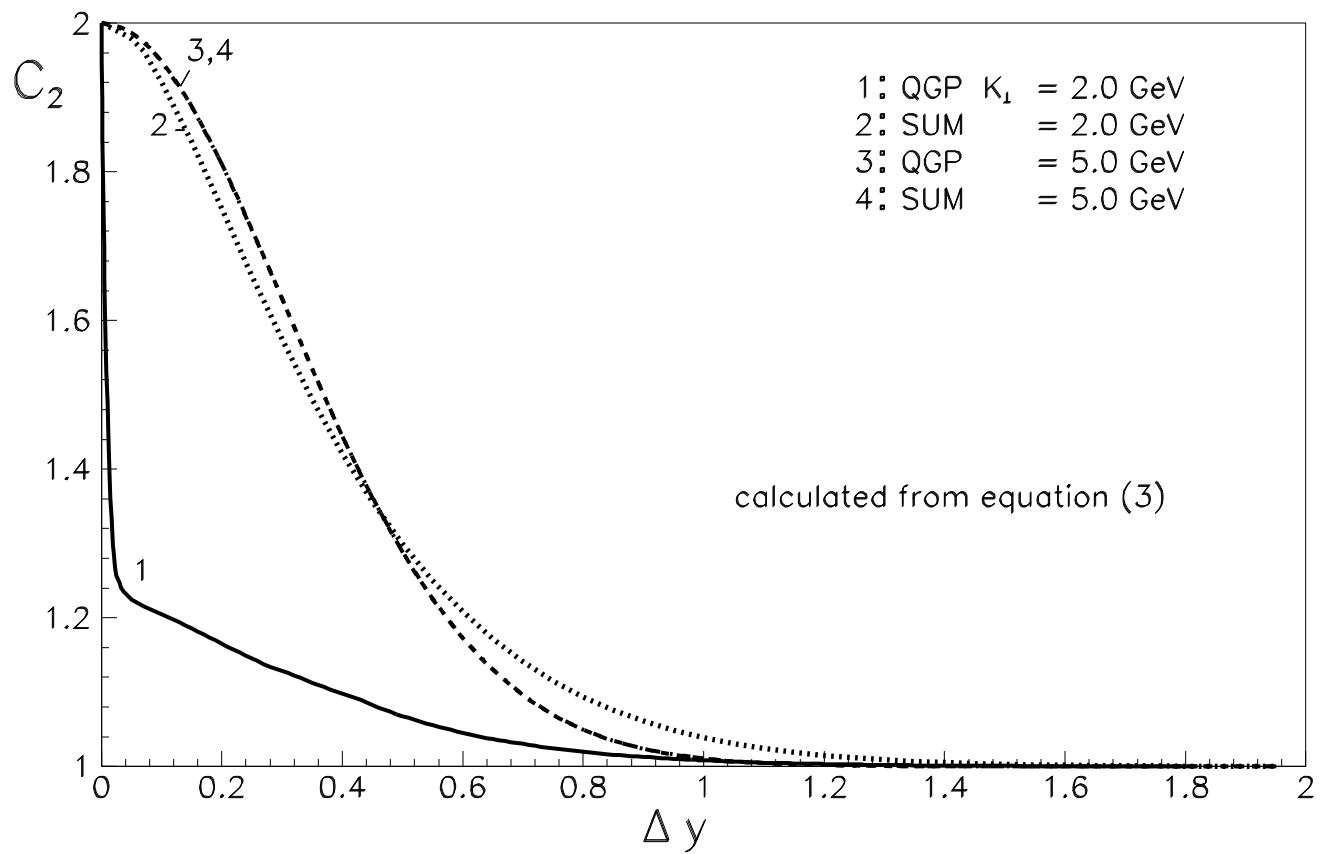


Fig. 4

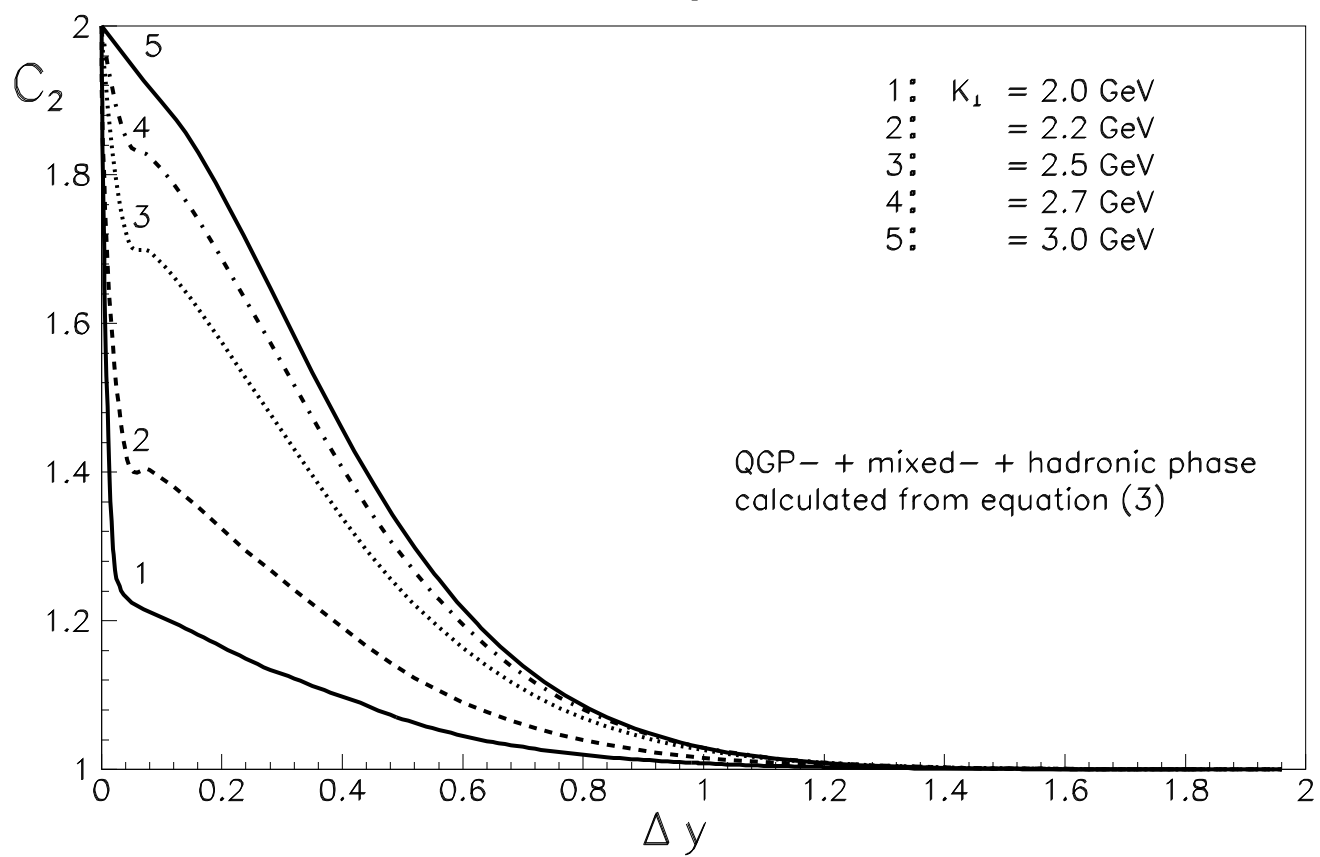


Fig. 5

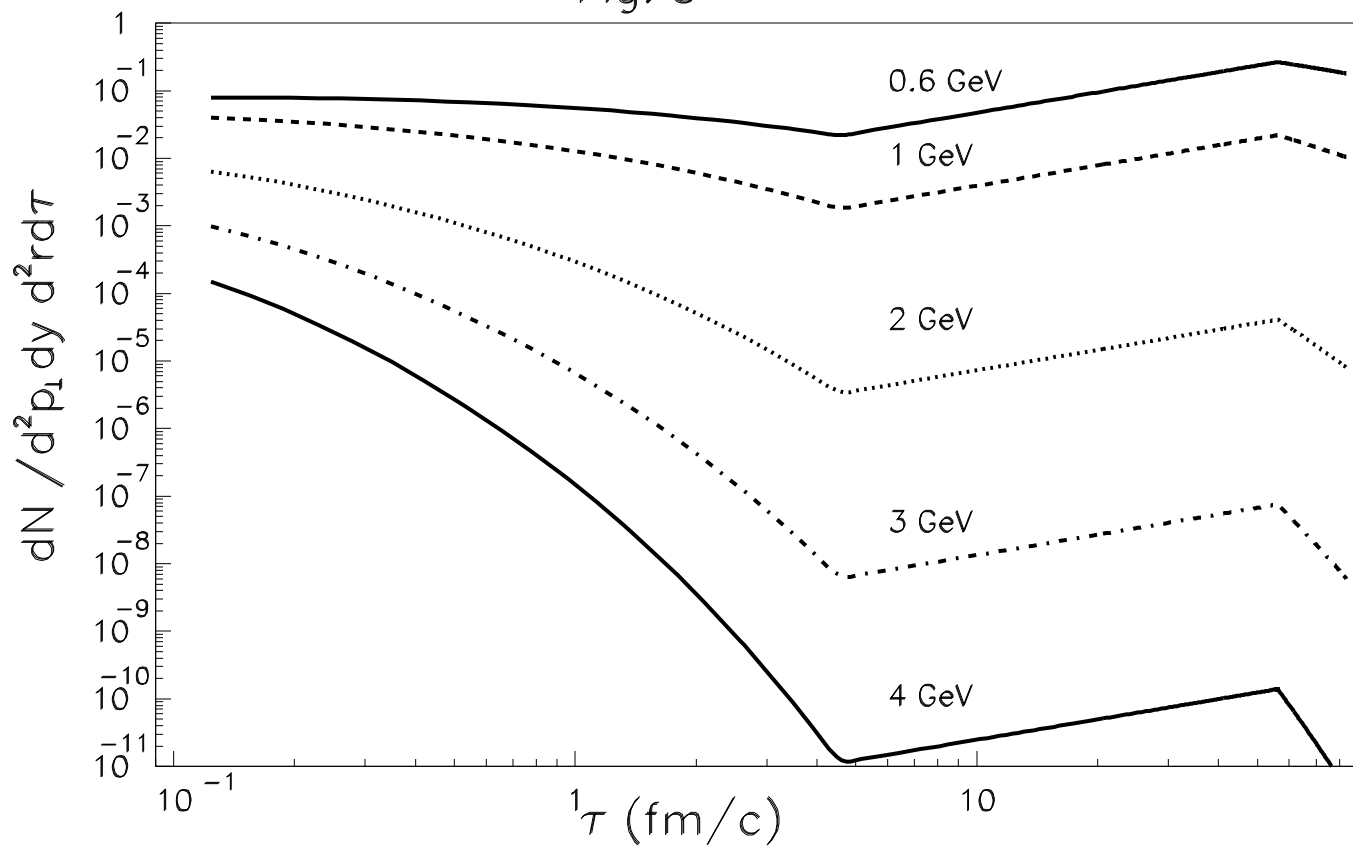


Fig. 6

